NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2013

MATHEMATICS: PAPER III

EXAMINATION NUMBER												

Time: 2 hours 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 16 pages and an Information Sheet of 2 pages (i – ii). Please ensure that your paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to FOUR (4) decimal digits where necessary.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.
8. The last pages can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	**Total**
Marks														

SECTION A

QUESTION 1

(a) The recursive formula for a sequence is given as $T_{n+1} = 3T_n + n; \quad n \geq 1; \quad n \in \mathrm{N}$.
The fifth term (T_5) of the sequence is 220.

Show all working to find the values of T_4 and T_3.

(4)

(b) The recursive formula for the sequence $2; 3; 4; 7; 15;$ … is given by:

$$T_{n+1} = T_n \times \frac{1}{2}T_{n-1} + p; \quad n \geq 2;\ n \in \mathrm{N}$$

(1) Find the value of *p*.

(2)

(2) Write down the value of T_6.

(1)
[7]

QUESTION 2

(a) Each of the digits 1; 1; 2; 3; 4; 7 is written on a separate card. The cards are then placed next to each other to make a 6 digit number.

(1) How many numbers start and end with the same digit?

_____

_____ (2)

(2) Find the probability that the number is 112347 or 743211.

_____

_____

_____ (4)

(b) *n* people (numbered 1; 2; 3; 4; 5; 6; 7; …; *n*) are arranged randomly in a line. Find the number of ways, in terms of *n*, that person 1 and person 2 are not standing next to one another. Leave your answer in unsimplified form.

_____

_____

_____ (4)
[10]

QUESTION 3

A study of numbers of male and female offspring in a certain population is being carried out. It is found that the first child in any family is equally likely to be male or female, but that for any subsequent offspring, the probability that they will be of the same sex as the previous child is $\frac{3}{5}$. No twins, triplets, etc. are possible.

(a) Find the probability that the first child of a family will be female.

_____ (1)

(b) Find the probability that the first two children of a family will be female.

_____ (1)

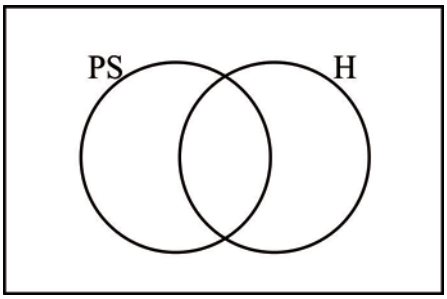
(c) Find the probability that a family will have two females followed by two males (in that order). Leave your answer in simplified faction form.

_____

_____ (2)
[4]

QUESTION 4

Half of the 150 matric learners at Van den Berg High School study Physical Science and 30% study History. Of those who study Physical Science 40% study History.



(a) Determine the number of the learners that study both Physical Science and History.

_____

_____ (2)

(b) If a learner is chosen at random, find the probability that the learner does not study either Physical Science or History.

_____ (2)
[4]

QUESTION 5

Events A and B are such that $P(A) = \frac{1}{4}$ and $P(\text{A or B}) = \frac{1}{3}$.

Find P(B) (as a simplified fraction) if:

(a) A and B are mutually exclusive events.

_____

_____ (2)

(b) A and B are not mutually exclusive events, but A and B are independent events.

_____

_____

_____ (4)
[6]

QUESTION 6

Mr Ryan is a retired teacher who supplements his pension by mowing lawns for customers who live in his neighbourhood.
As part of a review of his charges for this work, he measures the approximate areas (x) (in m^2) of a random sample of 12 of his customers' lawns and notes the time (y) in minutes, that it takes him to mow these lawns.

His results are shown in the table.

Area (x) (m^2)	360	120	845	602	1 190	530	245	486	350	1 005	320	250
Time (y) (minutes)	50	28	130	75	120	95	55	70	48	110	55	60

(a) Use your calculator to determine the equation of the least squares regression line. Give your answers correct to 4 decimal digits.

_____

_____ (5)

(b) Calculate the value of r, the correlation coefficient for the data, correct to 4 decimal digits.

_____ (2)

(c) Given that Mr Ryan charges a flat call out fee of R150, as well as R50 per half hour (or part thereof), estimate the charge for mowing a customer's lawn that has an area of 560 m^2.
(For example: 100 minutes would be taken as 2 hours)

_____

_____ (3)

(d) The local high school want Mr Ryan to mow their rugby field which is rectangular, 100 metres long by 70 metres wide. Should you use the regression equation found in (a) to calculate the time it would take to mow this area? Give a reason for your answer.

_____

_____ (2)
[12]

QUESTION 7

The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the employees of a certain company.

Daily travelling time x (in minutes)	Number of employees
$0 \leq x < 10$	20
$10 \leq x < 20$	35
$20 \leq x < 30$	30
$30 \leq x < 40$	10
$40 \leq x < 50$	5

(a) Circle the correct answer for the following questions:

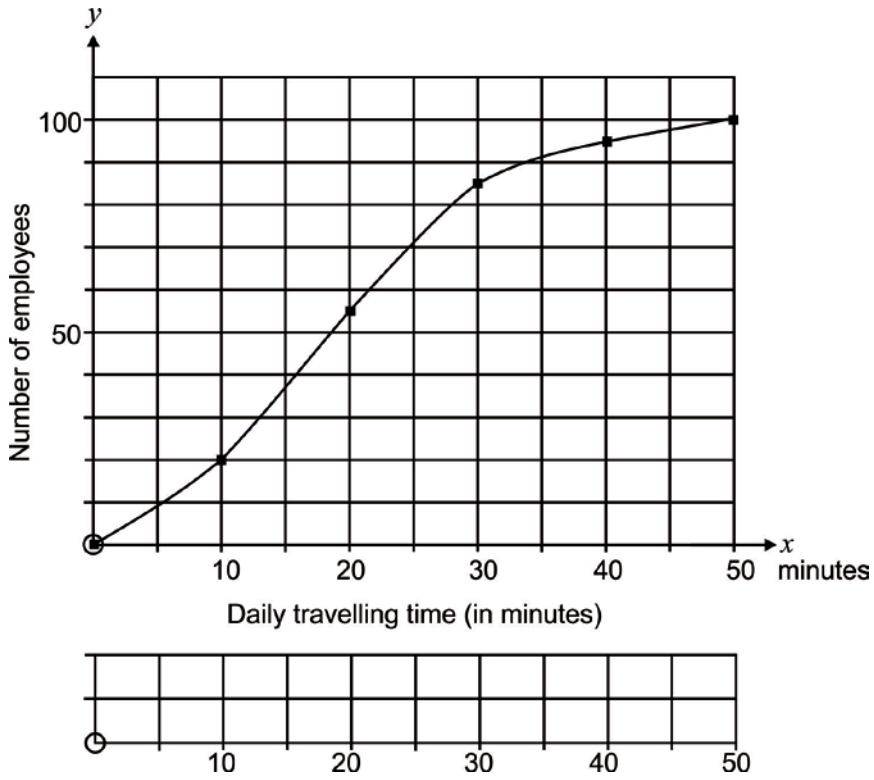
(1) The estimated mean time (in minutes) taken by employees is:

A 14,5 B 19,5 C 16,7 D 24,5 (3)

(2) The estimated standard deviation for the time (in minutes) is:

A 10,57 B 14,14 C 114,75 D 10,71 (2)

(b) An ogive was constructed from the given data.
Construct a box-and-whisker plot on the scaled axis below the ogive, to summarise the given data.



(3)

(c) State whether the following is TRUE or FALSE.

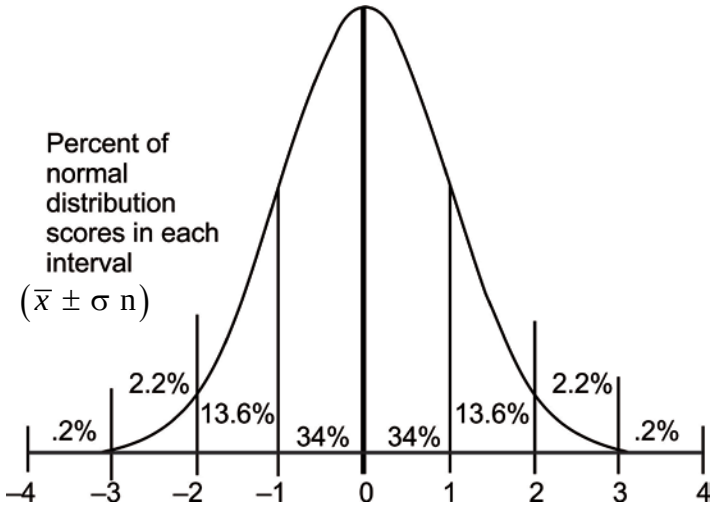
(1) The distribution of these travelling times is skewed positively. _____

(2) The inter-quartile range for this data is 25. _____

(3) 35 employees take less than 20 minutes. _____ (3)

[11]

QUESTION 8



A biologist has collected data on the heights of a particular species of cactus. He observes that 2,4% of the cacti are below 12 cm and 16% are above 17,22 cm in height. He assumes that the heights are normally distributed.

(a) Find the standard deviation of the distribution.

_____

_____

_____ (3)

(b) Find the mean of the distribution.

_____ (1)

(c) In a sample of 300 cacti of this species, estimate how many would be expected to be between 12 cm and 17,22 cm in height. Leave your answer rounded to the nearest whole number.

_____ (2)

[6]

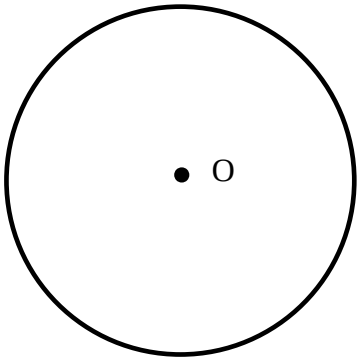
60 marks

SECTION B

QUESTION 9

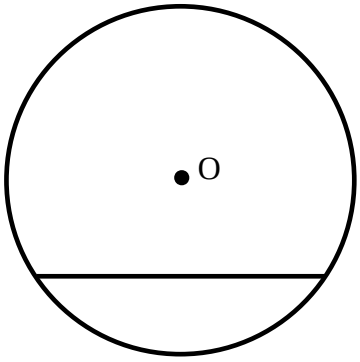
In each case below, you are given a statement and a reason that are true for the incomplete diagram. Complete the diagram, showing what was necessary so that the statement and the reason are true.

(a) Statement: $A\hat{O}B = 2A\hat{C}B$.
Reason: $\angle$ at centre equals $2 \times \angle$ at the circumference.



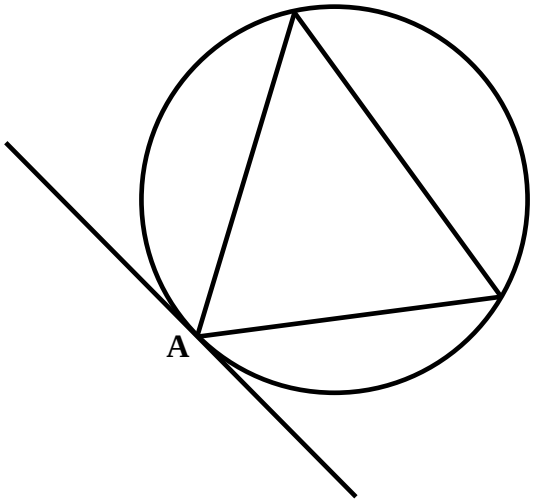
(1)

(b) Statement: $TS = SP$.
Reason: line from centre perpendicular to chord.



(1)

(c) Statement: $B\hat{A}D = \hat{T}$.
Reason: tan chord theorem.

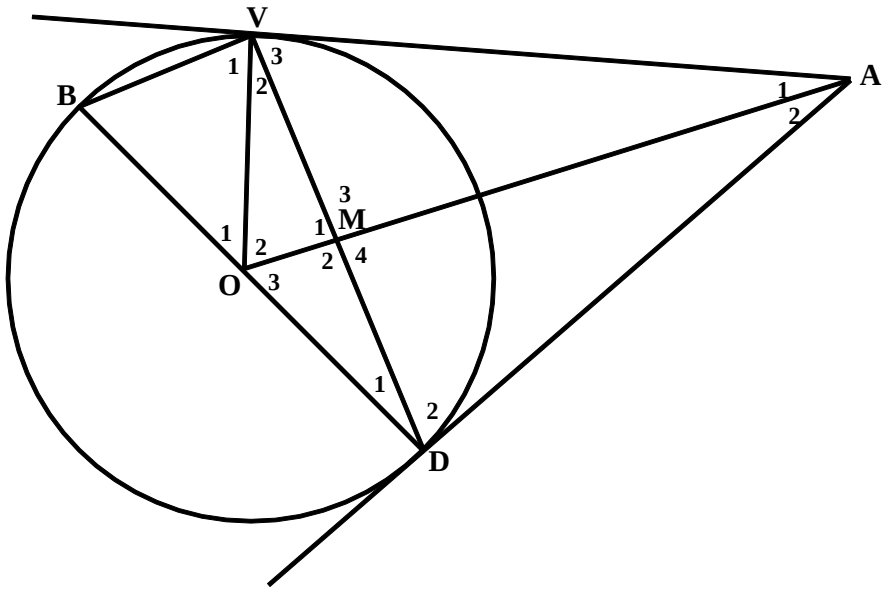


(1)
[3]

QUESTION 10

From a point A outside the circle, centre O, two tangents AD and AV are drawn. AO and VD meet in M. BOD is a diameter of the circle. BV and VO are drawn.

Let $\hat{A}_1 + \hat{A}_2 = 40^o$



(a) Complete the following table:

Statement	Reason
(1) $A\hat{D}O = 90°$	_____
(2) $B\hat{V}D = 90°$	_____

(2)

(b) Calculate, with reasons, the size of:

(1) $\hat{D}_1$

_____

_____

_____

_____

_____ (4)

(2) $\hat{O}_1$

_____

_____

_____

_____ (2)

(c) Prove, with reasons, that BV is parallel to OA, i.e. BV ∥ OA.

_____

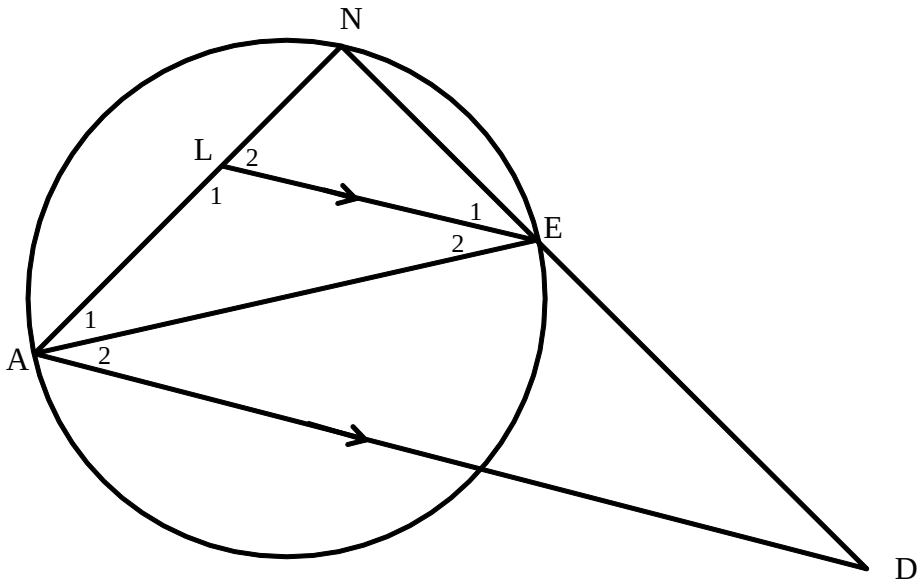
_____

_____ (3)

[11]

QUESTION 11

In the figure AE is a diameter of circle ANE. L is a point on AN and LE bisects $A\hat{E}N$.
Let $\hat{E}_1 = \hat{E}_2 = x$.
NE produced meets a line from A parallel to LE, in D.
Hence LE || AD.



(a) Complete the following table:

Statement	Reason
$\hat{E}_1 = \hat{D}$	_____
$\hat{E}_2 = \hat{A}_2$	_____
$\therefore AE = ED$	_____

(3)

(b) If NE = 12 units and the diameter of the circle is 20 units, calculate giving reasons:

(1) AN

_____

_____

_____

_____

_____ (3)

(2) AL

_____

_____

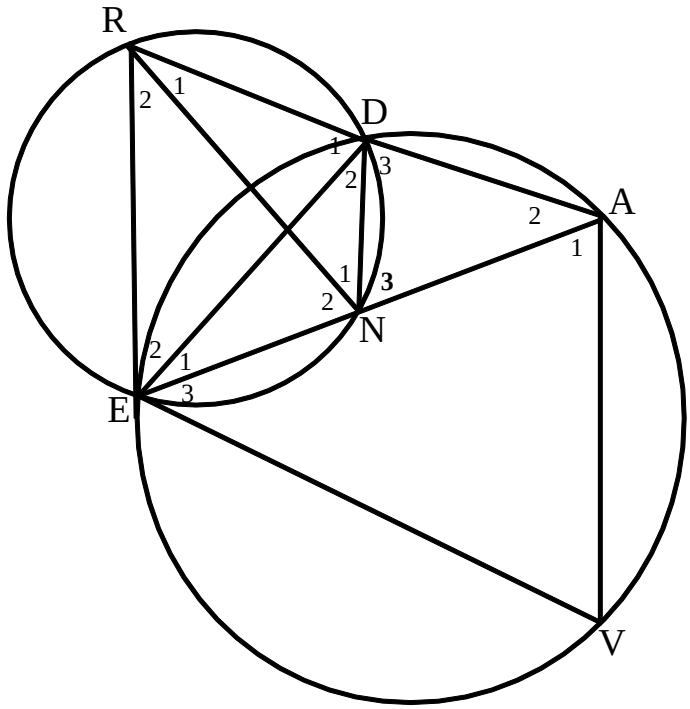
_____

_____ (4)

[10]

QUESTION 12

Two circles intersect at D and E. Chord RE of the smaller circle is a tangent to the larger circle at E. N is a point on the small circle.
EN and RD are produced to meet the bigger circle at A. RN, ED and DN are drawn.
V is a point on the larger circle and AV and EV are drawn.



(a) Complete the following table:

Statement	Reason
$\hat{N}_1 = \hat{E}_2$	_____ (1)
$\hat{E}_2 = \hat{A}_2$	_____ (1)
$\therefore \hat{N}_1 = \hat{A}_2$	

(b) Prove $\hat{D}_1 = \hat{E}_1 + \hat{E}_2$.

_____

_____

_____ (2)

(c) Prove, with reasons, that $\Delta EDR /// \Delta AER$.

_____

_____

_____

_____ (3)

(d) If $2AV = DR.AR$ and $ER = 3$ cm find the length of AV.

_____

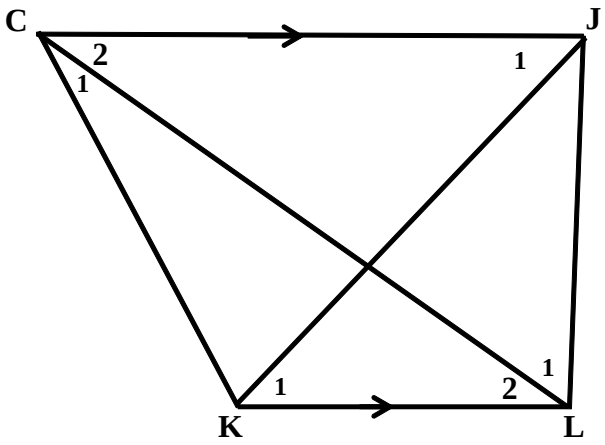
_____

_____ (3)

[10]

QUESTION 13

In the diagram, which is not drawn to scale, KLJC is a trapezium with KL ‖ CJ.
CK = 24 cm, KL = 8 cm, LJ = 12 cm, JC = 32 cm and KJ = 16 cm.



Determine the ratio $\frac{\text{Area of } \Delta KLJ}{\text{Area of } CKLJ}$.

[6]

40 marks

Total: 100 marks